Name: ______________________________

PRECALCULUS SUMMER PACKET

Directions: This packet is required if you are registered for Precalculus for the upcoming school year. The packet will be collected on the first day of school and given homework points. The topics in this packet are all skills that you have already seen, and it is necessary that you are comfortable working problems of these types upon entering Precalculus. Use the provided example problems and your notes from previous classes to assist in completion.

Functions

Operations with Functions

Example: *Find* $(f+g)(x)$, $(f-g)(x)$, $(f\cdot g)(x)$, $\left(\dfrac{f}{g}\right)(x)$ *for:* $f(x)=x^2+3x-4$ *and* $g(x)=3x-2$.

$$\begin{aligned}(f+g)(x)&=f(x)+g(x)\\&=(x^2+3x-4)+(3x-2)\\&=x^2+6x-6\end{aligned}$$

$$\begin{aligned}(f-g)(x)&=f(x)-g(x)\\&=(x^2+3x-4)-(3x-2)\\&=x^2+3x-4-3x+2\\&=x^2-2\end{aligned}$$

$$\begin{aligned}(f\cdot g)(x)&=f(x)\cdot g(x)\\&=(x^2+3x-4)(3x-2)\\&=x^2(3x-2)+3x(3x-2)-4(3x-2)\\&=3x^3-2x^2+9x^2-6x-12x+8\\&=3x^3+7x^2-18x+8\end{aligned}$$

$$\begin{aligned}\left(\frac{f}{g}\right)(x)&=\frac{f(x)}{g(x)}\\&=\frac{x^2+3x-4}{3x-2},\ x\neq\frac{2}{3}\end{aligned}$$

Practice: *Perform the following operations for:* $f(x)=2x+1$ *and* $g(x)=x-3$.

1. (f + g)(x)
2. (g – f)(x)
3. (f –g)(x)
4. (f · g)(x)
5. $\left(\dfrac{g}{f}\right)(x)$
6. $\left(\dfrac{f}{g}\right)(x)$

Evaluating Functions

Examples:

$$\begin{aligned}m(x)&=2x^2-x+2\\m(2)&=2(2)^2-(2)+2\\&=8\end{aligned}$$

$$\begin{aligned}r(x)&=-x^3+x+1\\r(-4)&=-(-4)^3+(-4)+1\\&=61\end{aligned}$$

Practice: *Evaluate for:* $c(x)=-16x^2-4x+2$, $h(x)=x^4-2x$, $k(x)=2x^3+5x^2$.

7. h(-2)
8. k(3)
9. $c\left(\dfrac{1}{2}\right)$

10. What does f(2)=5 mean?

Factoring

Examples: Factor.

GCF: $4x^3y + 6x^2y - 2x$

$= 2x(2x^2y + 3xy - 1)$

Difference of 2 Squares: $4x^2 - 9$

$= (2x)^2 - (3)^2$

$= (2x-3)(2x+3)$

Product/Sum: $x^2 + 2x - 15$

(or reverse FOIL) $= (x+5)(x-3)$

x (-15)	+ (2)
-1, 15	14
1, -15	-14
-5, 3	-2
5, -3	+2

By Grouping: $x^3 - 4x^2 + 2x - 8$

$= (x^3 - 4x^2) + (2x - 8)$

$= x^2(x-4) + 2(x-4)$

$= (x-4)(x^2+2)$

- *Use ( )+( ) to group*
- *Factor out the GCF of each group*
- *Factor out the common (_) out of each term*

Master Product: $2x^2 - 7x + 3$

(or just guess and check)

x 6	+ (-7)
2, 3	5
-2, -3	-5
1, 6	7
-1, -6	-7

$= 2x^2 \overbrace{-1x - 6x}^{-7x} + 3$

$= (2x^2 - 1x) + (-6x + 3)$

$= x(2x-1) + -3(2x-1)$

$= (2x-1)(x-3)$

- *In* $ax^2 + bx + c$ *form, multiply a x c*
- *Find 2 #'s that multiply to axc and add to b*
- *Replace the middle term*
- *Factor by grouping.*

Practice: Factor the following completely.

1. $5a^2b + 10ab^3$
2. $x^2 - 25$
3. $x^2 + 6x$
4. $1 - 9x^2$
5. $6x^3 - 9x^2 + 2x - 3$
6. $5x^2 - 20$
7. $x^2 - 8x + 15$
8. $5x^2 - 7x + 2$
9. $2x^2 - 2x - 24$
10. $6x^2 - x - 1$

Solving Quadratic Equations

***Examples:* Solve the following quadratic equations.**

a.) Solve by factoring:

$$x^2+2x-15=0$$
$$(x-3)(x+5)=0$$
$$x-3=0 \qquad x+5=0$$
$$\boxed{x=3 \qquad x=-5}$$

- *Factor the equation*
- *Set each factor = to 0.*
- *Solve for x.*

b.) Solve by using the quadratic equation:

$$2x^2-3x-6=3$$
$$2x^2-3x-9=0 \qquad a=2, b=-3, c=-9$$
$$x=\frac{+3\pm\sqrt{(-3)^2-4(2)(-9)}}{2(2)}$$
$$x=\frac{3\pm\sqrt{81}}{4}$$
$$x=\frac{3+9}{4}=\frac{12}{4}=3 \qquad x=\frac{3-9}{4}=\frac{-6}{4}=\frac{-3}{2} \qquad \boxed{x=3, \frac{-3}{2}}$$

- *Set up in $ax^2+bx=c=0$ form. and determine a,b,c*
- *Substitute into the quadratic equation:*

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

- *Simplify.*

c.) Solve by using the quadratic equation, simplify the radicals:

$$x^2-4x=8$$
$$x^2-4x-8=0 \qquad a=1, b=-4, c=-8$$
$$x=\frac{+4\pm\sqrt{(-4)^2-4(1)(-8)}}{2(1)}$$
$$x=\frac{4\pm\sqrt{48}}{2} \longrightarrow x=\frac{4\pm4\sqrt{3}}{2} \longrightarrow \boxed{x=2\pm2\sqrt{3}}$$

Practice: *Solve.*

1. $x^2+7x+12=0$

2. $5x^2=10x$

3. $2x^2+x=15$

4. $-x^2+2x+10=0$

5. $x^2-16=0$

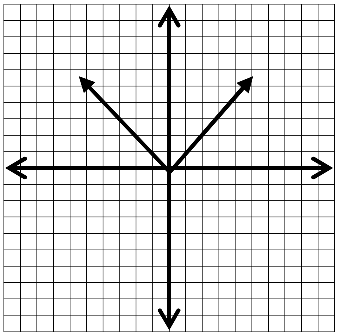
6. $2x^2-3x-3=0$

Shifting Graphs

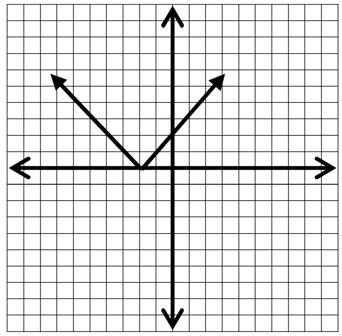
Example:
Be familiar with the graphs of common functions.
Clearly shift 3 points.

- Vertical shift *c* units upward: *h(x)=f(x)+c*
- Vertical shift *c* units downward: *h(x)=f(x)-c*
- Horizontal shift *c* units to the right: *h(x)=f(x-c)*
- Horizontal shift *c* units to the left: *h(x)=f(x+c)*

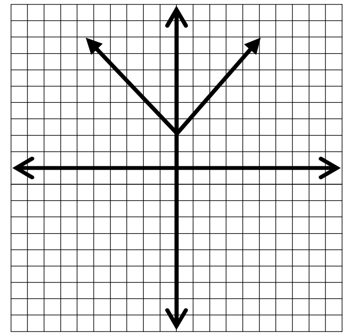
$f(x)=|x|$



$f(x)=|x+2|$

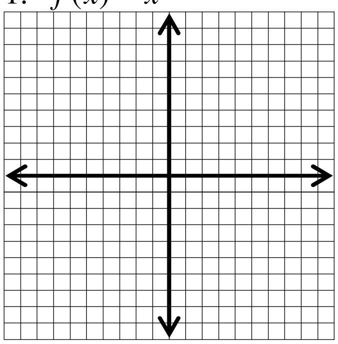


$f(x)=|x|+2$

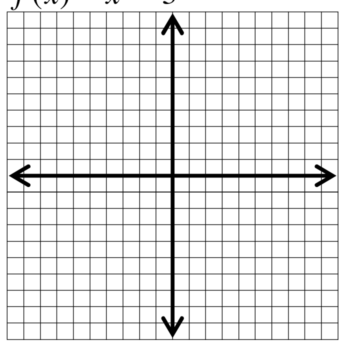


Practice:

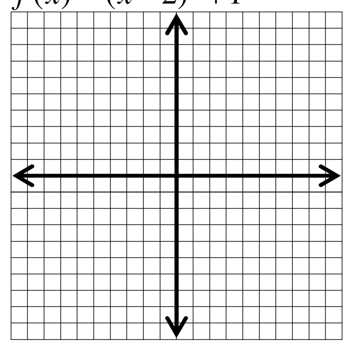
1. $f(x)=x^2$



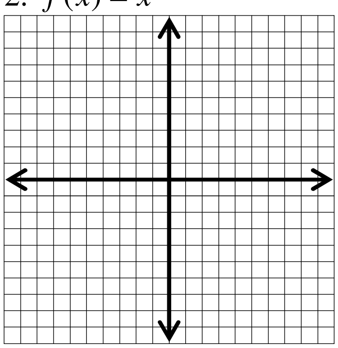
$f(x)=x^2-3$



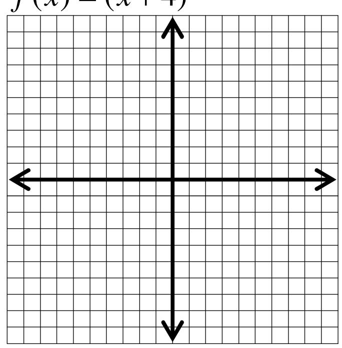
$f(x)=(x-2)^2+1$



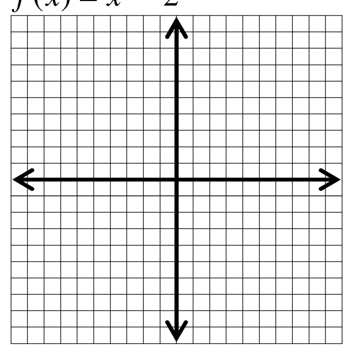
2. $f(x)=x^3$



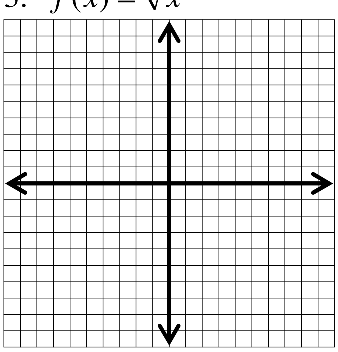
$f(x)=(x+4)^3$



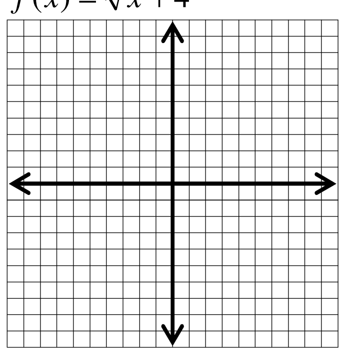
$f(x)=x^3-2$



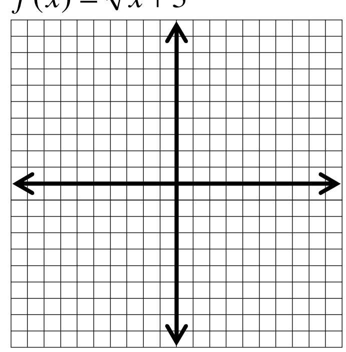
3. $f(x)=\sqrt{x}$



$f(x)=\sqrt{x}+4$



$f(x)=\sqrt{x+3}$



Piecewise functions: Graphing and Evaluating

Piecewise functions are composed of two or more functions.

Given the function: $f(x)=\begin{cases}2x+3, & x\le 1\\ -x+4, & x>1\end{cases}$

To the left of $x=1$, the graph is the line $y=2x+3$.

To the right of $x=1$, the graph is the line $y=-x+4$

Evaluate:

$f(-3)=2(-3)+3=-3$
$f(-2)=2(-2)+3=-1$
$f(-1)=2(-1)+3=1$
$f(0)=2(0)+3=3$
$f(1)=2(1)+3=5$

⇧

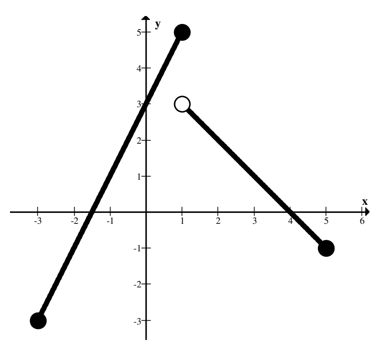
Use the first piece

$f(2)=-2+4=2$
$f(3)=-3+4=1$
$f(4)=-4+4=0$
$f(5)=-5+4=1$

⇧

Use the second piece

To graph, use the points from above



x	y
-3	-3
-2	-1
-1	1
0	3
1	5

x	y	
2	2	
3	1	
4	0	
5	1	
1	3	**Open Dot**

** You must evaluate the break point in each piece! One point will be a closed dot and the other will be an open dot,

Evaluate and graph.

1. $f(x)=\begin{cases}3x+3, & x<0\\ 5-x, & x\ge 0\end{cases}$

x	y

2. $f(x)=\begin{cases}x+1, & x\le -1\\ 2x+2, & x>-1\end{cases}$

x	y

Domain and Range

Given a function y = f(x), the Domain of the function is the set of permissible inputs (x-values) and the Range is the set of resulting outputs (y-values). Domains can be found algebraically; ranges are often found graphically. Domains and Ranges are sets. Therefore, you must use proper set notation.

When finding the domain of a function, ask yourself what values can't be used. Your domain is everything else. There are simple basic rules to consider:

- The domain of all polynomial functions is the set of real numbers.
- Square root functions cannot contain a negative underneath the radical. Set the expression under the radical greater than or equal to zero and solve for the variable. This will be your domain.
- Rational functions can not have zeros in the denominator. Determine which values of the input cause the denominator to equal zero, and set your domain to be everything else.

Examples: Find the domain of the function algebraically

.

1. $f(x) = \dfrac{4}{x-2}$ Domain: $x \neq 2$

2. $f(x) = \dfrac{3}{x^2 - 4x} = \dfrac{3}{x(x-4)}$ Domain: $x \neq 0, 4$

3. $f(x) = \dfrac{x}{x^2 - 9} = \dfrac{x}{(x+3)(x-3)}$ Domain: $x \neq -3, 3$

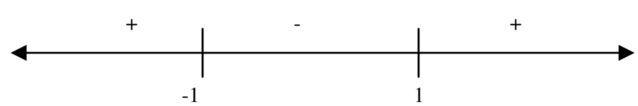
4. $f(x) = \sqrt{x-9}$ Domain: $x \geq 9$

5. $f(x) = \sqrt{4-x}$ Domain: $x \leq 4$

6. $f(x) = \sqrt{x^2 - 1}$ Set Radicand equal to zero and solve.

$$x^2 - 1 = 0$$
$$(x+1)(x-1) = 0$$
$$x = 1 \ \ \& \ \ x = -1$$

Now use a number line and decide where values will be positive



Domain: $x \leq -1 \ \ \& \ \ x \geq 1$

** A number line can be used for all square root equations by testing a number in each region.**

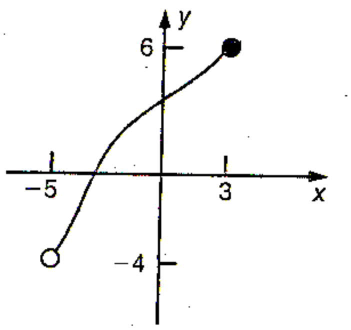
Domain and Range Continued

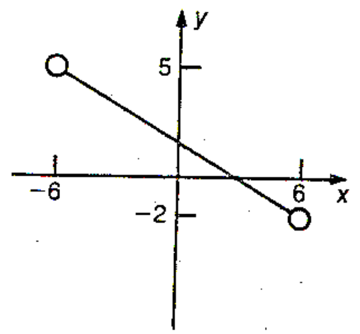
Identifying the domain and range from the graph is easy. For domain values, take the x-values from left to right. For range values, take the y-values from bottom to top.

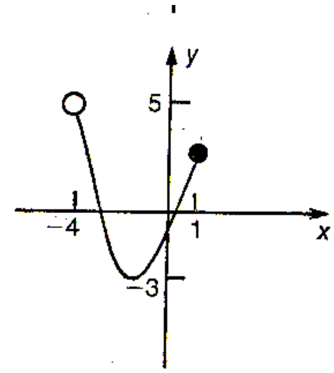
1. Domain: $-5 < x \le 3$
   Range: $-4 < y \le 6$

Domain: $-4 < x \le 1$
Range: $-3 \le y < 5$

Domain: $-6 < x < 6$
Range: $-2 < y < 5$







Practice: Identify the domain for each function.

1. $f(x) = \dfrac{3x}{x+2}$

2. $g(x) = \dfrac{x-1}{3x+2}$

3. $h(x) = \sqrt{2x-4}$

4. $f(x) = x^2 + 3x - 4$

5. $g(x) = \sqrt{5-x}$

6. $h(x) = \dfrac{3}{1+x^2}$

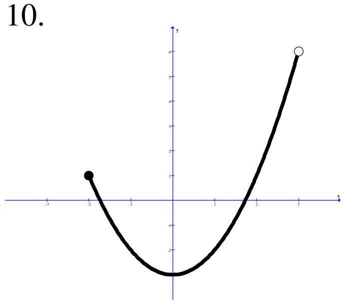
7. $f(x) = \dfrac{3}{x^2 - x - 6}$

8. $g(x) = \sqrt{x^2 + 9x + 20}$

9. $h(x) = \dfrac{x}{\sqrt{x^2 - 4}}$

Practice: Identify the domain and range for each function.

10.



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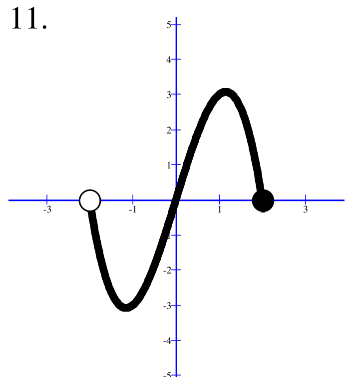

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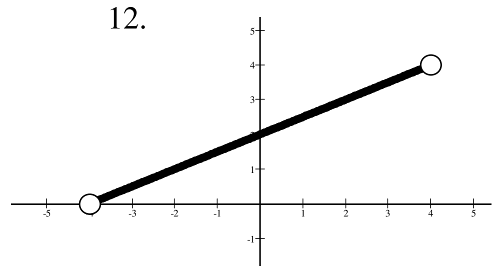

Unit Circle